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SOME RELATION THEORETIC FIXED POINT THEOREMS FOR RELATIONAL R-CONTRACTION MAPPINGS IN THE EUCLIDEAN SPACE $\mathbb{R}^n$

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Abstract. In this paper we prove a couple of relation theoretic fixed point theorems for linear partial **R**-contraction mappings $\mathcal{T}$ in the Euclidean space $\mathbb{R}^n$ via calculus method by using the max/mini principle. It is shown that though our approach is different from the usual one, we are not far away from the usual constructive method. Actually one can take an arbitrary point $x_0 \in \mathbb{R}^n$ which is relatable to $\mathcal{T}x_0$ and defines a sequence $\{x_n\}$ of iterates of the mapping under consideration, then it is shown that the sequence $\{x_n\}$ converges to a unique relatable fixed point relatonically and geometrically.

Keywords. Euclidean space; Max/mini principle; Binary relation; Contraction mappings; Fixed point theorem.

AMS (MOS) subject classification: 54H25, 47H10.

1 Introduction

Let $\mathbb{R}$ denote the real line and let $\mathbb{R}^n$ denote the n-dimensional Euclidean space with norm $\|x\|$. A binary relation or simply relation $\mathbf{R}$ in $\mathbb{R}^n$ is a subset of the Cartesian product $\mathbb{R}^n \times \mathbb{R}^n$ which is full if $\mathbf{R} = \mathbb{R}^n \times \mathbb{R}^n$ and which is empty if $\mathbf{R} = \emptyset$. $\mathbf{R}$ is called partial relation or proper relation if $\emptyset \neq \mathbf{R} \neq \mathbb{R}^n \times \mathbb{R}^n$. We introduce a partial transitive relation $\mathbf{R}$ in $\mathbb{R}^n$ and two elements a and b in $\mathbb{R}^n$ are said to be relatable if they are related to each other under the relation $\mathbf{R}$. In this case we write $a\,\mathbf{R}\,b$ or $(a, b) \in \mathbf{R}$. The partial relation $\mathbf{R}$ is said to be transitive if $(x, y) \in \mathbf{R}$ and $(y, z) \in \mathbf{R}$, then $(x, z) \in \mathbf{R}$. A set of all elements related under the transitive relation $\mathbf{R}$ is called totally related or **R**-chain and it is denoted by $C_{\mathbf{R}}$. A sequence $\{x_n\}$ of points in $\mathbb{R}^n$ is called relatonic if $(x_n, x_{n+1}) \in \mathbf{R}$ and inverse **relatonic** or **term-wise related** if $(x_{n+1}, x_n) \in \mathbf{R}$ for each $n \in \mathbb{N}$.

We say a subset S of $\mathbb{R}^n$ is **R**-regular or PR-closed or simply closed with respect to the relation $\mathbf{R}$, if $\{x_n\}$ is a relatonic (resp. inverse relatonic) sequence of points in $\mathbb{R}^n$ converging to a point x^*, then $(x_n, x^*) \in \mathbf{R}$ (resp.