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A HOMOTOPY APPROACH FOR APPROXIMABLE MAPS

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Abstract. Using homotopy ideas and essential maps we present some new Leray–Schauder type alternatives for approximable type maps.

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1 Introduction

In this paper we begin by establishing some fixed point results for compact approximable maps defined on Hausdorff locally convex topological vector spaces. We will then use the notions of homotopy, essential maps, and the above fixed point theory to obtain new Leray–Schauder alternatives for compact approximable maps. In the process of constructing these results we will discuss how a topological transversality type theorem [5, 9] might arise for the class (or a subclass) of compact approximable maps.

We first present the class of maps from the literature considered in this paper. Let X and Y be subsets of Hausdorff topological vector spaces E_1 and E_2 respectively and $F : X \to K(Y)$ (nonempty compact subsets of Y). Given two open neighborhoods U and V of the origins in E_1 and E_2 respectively, a (U, V)–approximate continuous selection ([1, 2, 3, 4, 10]) of $F : X \to K(Y)$ is a continuous function $s : X \to Y$ (i.e. $s \in C(X, Y)$) satisfying

$$s(x) \in (F[(x + U) \cap X] + V) \cap Y \quad \text{for every} \quad x \in X.$$

We say $F : X \to K(Y)$ is approachable (and we write $F \in A(X, Y)$) if it has a (U, V)–approximate continuous selection for every open neighborhood U and V of the origins in E_1 and E_2 respectively. We say $F : X \to K(Y)$ is approximable (and we write $F \in AP(X, Y)$) if its restriction $F|_\Omega$ to any compact subset Ω of X admits a (U, V)–approximate continuous selection for