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SPECTRAL GAP CONDITIONS FOR INERTIAL MANIFOLDS FOR SEMILINEAR PARABOLIC EQUATIONS WITH NON-INSTANTANEOUS IMPULSES

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Abstract. In this paper, using the Lyapunov–Perron method, we establish an explicit spectral gap condition ensuring the existence of an inertial manifold for a class of semilinear parabolic equations with non-instantaneous impulses.

Keywords. inertial manifold, parabolic equation, non-instantaneous impulse, Lyapunov-Peron method, spectral gap condition.

AMS (MOS) subject classification: 35B42, 35B35, 35R12.

1 Introduction and statement of the main result

The theory of instantaneous impulsive differential equations provides a mathematical framework for modeling processes that undergo abrupt changes in state at specific moments in time. Such processes frequently arise in a wide range of disciplines, including physics, chemistry, biology, population dynamics, engineering, and economics, where sudden transitions or discontinuities are inherent to the phenomena being studied. A distinguishing characteristic of instantaneous impulsive differential equations lies in their capacity to model processes and phenomena influenced by discrete, short-duration perturbations occurring over negligible time intervals relative to the overall evolution of the system. These equations are particularly well-suited for capturing dynamics in which the impulses are abrupt and effectively instantaneous within the framework of the mathematical model, see, e.g., [4, 12, 8, 13], and the cited references.

Nevertheless, it has been observed that models based on instantaneous impulses are insufficient to capture certain aspects of the dynamics involved in pharmacotherapeutic processes. As studied by Hernández and O'Regan [20] even under simplified assumptions regarding a person's hemodynamic equilibrium, the administration of drugs into the bloodstream and their subsequent absorption by the body constitute gradual and continuous processes. This motivates the interpretation of such events as impulsive actions that begin abruptly but persist over a finite time interval. These are referred to as *non-instantaneous impulses* in the context of mathematical modeling. In this context, differential equations with non-instantaneous impulses have increasingly