

ULAM–HYERS–RASSIAS STABILITY OF NONLINEAR HAHN VOLTERRA INTEGRO-DIFFERENCE EQUATIONS

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Abstract. This paper is confined to obtaining sufficient conditions for the existence, uniqueness, and stability of solutions to a class of first-order nonlinear Volterra integro-difference equations generated by the Hahn difference operator $\Delta_{q,\omega}$, which unifies and generalizes the classical forward difference operator and the Jackson q -difference operator. The equation under study combines a linear term, a Volterra-type integral term with kernel K , and a general nonlinear perturbation $\mathcal{G}$ involving a composed nonlinear operator μ , thereby extending previous linear Hahn difference equations to a substantially broader nonlinear integro-difference setting. Finally, a concrete example involving a translation operator and a Lipschitz-type nonlinearity is presented to illustrate how the abstract hypotheses can be verified in practice, and to demonstrate the applicability of the theoretical results. The paper concludes by outlining directions for future work on Ulam-type stability of higher-order nonlinear Hahn difference equations.

Keywords. Hahn difference equations, Ulam–Hyers stability, Ulam–Hyers–Rassias stability.

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1 Introduction and Preliminaries

Hahn defined his operator $\Delta_{q,\omega}$ to be

$$\Delta_{q,\omega}h(\tau) = \frac{h(q\tau + \omega) - h(\tau)}{\tau(q - 1) + \omega}$$

for every τ with $\tau \neq \omega_0 = \frac{\omega}{1-q}$ and $\Delta_{q,\omega}h(\omega_0) = h'(\omega_0)$ provided that $h'(\omega_0)$ exists in the ordinary sense. Here, $q \in (0, 1)$ and $\omega > 0$. We assume that $\mathcal{I}_{\mathcal{T}} = [\omega_0, \mathcal{T}]$, $\mathcal{T} > \omega_0$, $\sigma(t) = qt + \omega : \mathcal{I}_{\mathcal{T}} \rightarrow \mathcal{I}_{\mathcal{T}}$ and h is an arbitrary function