Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **33 (2026)** 241-267

A NEW NUMERICAL SCHEME FOR THE FRACTIONAL OPERATOR WITH MITTAG-LEFFLER KERNEL AND ITS APPLICATION TO NEURAL NETWORK SYSTEMS

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Abstract. In this article, we introduce a generalized numerical differentiation formula tailored to solve fractional differential equations involving the recently proposed new generalized Katugampola fractional derivative of order $\alpha \in [0, 1]$. This proposed fractional derivative combines the structural features of the Katugampola fractional operator with the non-singular Mittag-Leffler kernel model, offering enhanced modeling capabilities for memory-dependent systems. To numerically approximate the proposed derivative, we develop a linear interpolation-based scheme that utilizes two successive points, $\left(t_{j-1}^p, y\left(t_{j-1}^p\right)\right)$ and $\left(t_j^p, y\left(t_j^p\right)\right)$, over each sub-interval $\left[t_{j-1}^p, t_j^p\right]$ $(j \geq 1)$. We rigorously analyze the existence and uniqueness of the solution, derive the absolute error and establish the theoretical order of convergence. The accuracy and efficiency of the proposed method were validated using two benchmark problems, and showed excellent agreement with the exact solutions. Furthermore, we demonstrate the applicability of the method by modeling the dynamics of fractional-order neural network systems governed by the proposed derivative.

Keywords. Fractional derivative, Mittag-Leffler kernel, Laplace transform, Generalized predictor-corrector, Neural network systems.

AMS (MOS) subject classification: 26A33, 34A08, 65L20, 92B20

1 Introduction

Fractional calculus generalizes the concept of integer-order differentiation and integration to non-integer fractional orders. It has emerged as a powerful tool for modeling memory and hereditary properties in various scientific domains. Its applications include anomalous diffusion, viscous elasticity, electrical circuits, control systems, and neural networks [1–3]. Classical definitions of fractional derivatives, including the Riemann-Liouville and Caputo types, are widely used but have singular kernels that can hinder numerical stability and physical interpretability [4].