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INTEGRABLE QUADRATIC DIFFERENTIAL SYSTEMS WITH A STRONG FOCUS

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Abstract. We study integrable quadratic polynomial differential systems in the plane having a strong focus at the origin. For the families previously obtained in the classification of Giné and Llibre, we compute explicit Darboux-type first integrals and determine the corresponding global phase portraits in the Poincaré disc. The main contribution of the paper is the complete description of the global dynamics, including the behavior at infinity, for all integrable quadratic systems with a strong focus. In particular, we show that these systems fall into a finite number of topological phase portraits.

Keywords. focus, quadratic system, first integral, Darboux integrability, global phase portraits.

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1 Introduction and statement of the main results

Let $\mathbb{R}[x, y]$ be the ring of the polynomials in the variables x and y with coefficients in $\mathbb{R}$. We consider a system of polynomial differential equations or simply a *polynomial differential system* in $\mathbb{R}^2$ defined by

$$\dot{x} = P(x, y), \qquad \dot{y} = Q(x, y), \tag{1}$$

where $P, Q \in \mathbb{R}[x, y]$ and the dot denotes derivative with respect to the independent variable t. We say that the maximum of the degrees of the polynomials P and Q is the *degree* of system (1). Usually a *quadratic polynomial differential system of degree* 2 is denoted simply as a *quadratic system.*

Sometimes we shall talk about the *quadratic vector field* $\mathcal{X}$ associated to the quadratic system (1) which is

$$\mathcal{X} = P(x, y)\frac{\partial}{\partial x} + Q(x, y)\frac{\partial}{\partial y}.$$