

## CONTINUATION PRINCIPLES FOR MAPS WITH UPPER SEMICONTINUOUS SELECTIONS

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**Dedicated to the memory of my friend Ravi P. Agarwal**

**Abstract.** In this paper we present some principles for maps which have upper semicontinuous Kakutani or acyclic type selections.

**Keywords.** Fixed points, set-valued maps, homotopy,  $d$ -essential maps.

**AMS (MOS) subject classification:** 47H10, 54H25

### 1 Introduction

This paper considers general classes of maps (these include  $KLU$  [5] and  $HLPY$  [6] maps) which have a selection property and we present continuation principles based on the notion of a  $d$ -essential map (this map gives an estimate on the fixed point set of the map). In applications one is usually interested in the existence of a fixed point so one usually considers  $d$  as in Remark 2.9(ii).

In this paper we will make use of Kakutani and acyclic maps. An upper semicontinuous map  $\Phi : X \rightarrow CK(Y)$  is said to be Kakutani and we write  $\Phi \in Kak(X, Y)$ ; here  $X$  is a Hausdorff topological space,  $Y$  is a Hausdorff topological vector space and  $CK(Y)$  denotes the family of nonempty, convex, compact subsets of  $Y$ . Let  $X$  and  $Z$  be subsets of Hausdorff topological spaces and let  $F : X \rightarrow K(Z)$  i.e.  $F$  has nonempty compact values. Recall a nonempty topological space is said to be acyclic if all its reduced Čech homology groups over the rationals are trivial. We will also consider maps  $F : X \rightarrow Ac(Z)$  i.e.  $F : X \rightarrow K(Z)$  with acyclic values (i.e.  $F$  has nonempty acyclic compact values). We say  $F \in AC(X, Z)$  (i.e.  $F$  is an acyclic map) if  $F : X \rightarrow Ac(Z)$  is upper semicontinuous.

The classes of maps in this paper are motivated from the following maps. Let  $Z$  be a subset of a Hausdorff topological space  $Y_1$  and  $W$  a subset of