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ENERGY ESTIMATES AND NON-TRIVIAL SOLUTIONS FOR A NON-LINEAR PARTIAL DISCRETE DIRICHLET PROBLEM WITH P-LAPLACIAN

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Abstract. The purpose of this paper is to investigate the existence of non-trivial solutions and derive energy estimates for a non-linear discrete Dirichlet boundary value problem involving the p-Laplacian and a real parameter λ. Under appropriate assumptions on the nonlinear term, we establish the existence of open intervals for the parameter λ within which non-trivial weak solutions exist. The analysis is conducted using variational methods and critical point theory, with the main tools being a local minimum theorem for differentiable functionals and the Mountain Pass Theorem. Several examples are provided to illustrate the applicability of the theoretical results.

Keywords. discrete non-linear boundary value problems, p-Laplacian, non-trivial solution, energy estimates.

AMS (MOS) subject classification: 39A10, 39A14, 39A70, 47J30.

1 Introduction

We study the following non-linear partial discrete Dirichlet problem:

$$\begin{cases} -\Delta_1(\phi_p(\Delta_1 u(i-1,j))) - \Delta_2(\phi_p(\Delta_2 u(i,j-1))) = \lambda f((i,j),u(i,j)), \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (i,j) \in [1,L]\times[1,J], \\ u(i,0) = u(i,J+1) = 0,\ i \in [0,L+1], \\ u(0,j) = u(L+1,j) = 0,\ j \in [0,J+1], \end{cases} \tag{1.1}$$

where two forward difference operators Δ_1 and Δ_2 are defined by $\Delta_1 u(i-1,j) = u(i,j) - u(i-1,j)$ and $\Delta_2 u(i,j-1) = u(i,j) - u(i,j-1)$ respectively and λ is a positive parameter, the p-Laplacian ϕ_p is defined by $\phi_p(t) = |t|^{p-2}t, 1 < p < \infty$ and $f : [1,L]\times[1,J]\times\mathbb{R} \to \mathbb{R}$ is continuous, where $[1,L] = \{1,2,...,L\}$ and $[1,J] = \{1,2,...,J\}$.