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ON DOUBLE ORDER FRACTIONAL BOUSSINESQ-BURGERS EQUATIONS WITH CUBIC NONLINEARITY

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Abstract. This paper investigates a singularly perturbed time-fractional coupled Boussinesq-Burger system within the scope of conformable derivatives. The proposed problem involves double-order time derivatives and dissipation terms, representing a generalization of the classical Boussinesq-Burger equations. The system is formulated using Khalil's conformable derivative definition, which maintains important properties while extending to fractional orders. The singular perturbation approach allows analysis of boundary layer behavior and temporal dynamics in fractional nonlinear systems, capturing complex physical phenomena including wave propagation in viscoelastic media and preserving a purely local description of the dynamics.

Keywords. Singular perturbation, time-fractional derivatives, conformable derivative, Boussinesq–Burger system.

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1 Introduction

Nonlinear evolution equations have long been essential tools for modeling complex real-word phenomena across various scientific disciplines [8, 21, 26]. Among these, the Boussinesq and Burgers equations represent two fundamental models in mathematical physics. The Boussinesq equation, originally derived by J. Boussinesq in the 19th century to describe shallow water waves [5], captures the delicate balance between nonlinearity and dispersion that characterizes soliton propagation [20, 24, 25]. Meanwhile, the Burgers equation serves as an important model for understanding nonlinear advection and dissipation processes [6], finding applications in fluid dynamics, gas dynamics, and traffic flow [4, 9, 18, 19].
Singular perturbation theory provides important analytical techniques for