POSITIVE SOLUTIONS FOR A CLASS OF NONLINEAR SEMIPOSITONE NONLOCAL FRACTIONAL BOUNDARY VALUE PROBLEM WITH SINGULARITIES BOTH ON TIME AND SPACE VARIABLE

S. Panigrahi[1] and Raghvendra Kumar[2]

[1,2]School of Mathematics & Statistics, University of Hyderabad Hyderabad- 500046 INDIA

Abstract. In this paper, an attempt has been made to establish the existence and multiplicity of positive solution for a class of nonlinear fractional differential equation subject to the Riemann-Stieltjes type boundary conditions by using different height function of the nonlinear term on special bounded sets by using Guo-Krasnosel'skii fixed point theorem of cone expansion-compression. The nonlinear term may change sign and may have singularitiy in the space variable. Finally, we have shown the importance of our results through examples.

Keywords: Continuous fractional calculus, Boundary value problem, Green's function, Positive solutions, Krasnosel'kii fixed point theorem in cone, Banach space.

AMS Subject Classification: Primary 34A08, Secondary 45G05.

1   Introduction

We will investigate the existence of positive solution for the following fractional differential equation

$$D_{0+}^{\alpha} u(t) + \lambda g(t) f(t, u) = 0, \quad t \in (0, 1), \tag{1.1}$$

with the boundary conditions

$$u(0) = u'(0) = u''(0) = \cdots = u^{(n-2)}(0) = 0, D_{0+}^{\beta} u(1) = \int_0^1 a(t) D_{0+}^{\beta} u(t) dA(t), \tag{1.2}$$

with $\alpha \in \mathbb{R}$, $\alpha \in (n-1, n]$, $n \in \mathbb{N}, n \geq 3, \beta \in (0, 1]$, $A : [0, 1] \to \mathbb{R}$ is a function of bounded variation, dA is a signed measure, $a \in C((0, 1), (0, \infty)) \cap L^1(0, 1)$ and $a(t)$ may have singularity at $t = 0$ and/or $t = 1$, the nonlinearity f may change sign and may be singular at $t = 0$ and/or $t = 1$ and also in the space variable $u = 0$, $g \in C((0, 1), [0, \infty))$ and may be singular at $t = 0$ and/or $t = 1$ and the parameter

---

[1]Corresponding author, Email: panigrahi2008@gmail.com
[2]Email: raghvendrakumar0802@gmail.com ; This work was supported by University Grant Commission, New Delhi, India.