EXISTENCE AND UNIQUENESS OF SOLUTION OF FRACTIONAL ORDER BOUNDARY VALUE PROBLEM WITH MIXED BOUNDARY CONDITION VIA A NEW THREE STEPS ITERATION PROCESS

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Abstract. The present paper discusses the existence and uniqueness of a nonlinear differential equation of fractional order involving the Caputo fractional derivative with the mixed boundary condition using a new three-step iteration process. Furthermore, the study of various properties such as dependence on boundary data, the closeness of solutions, and dependence on parameters and functions are discussed. In the last part, an example is used to demonstrate each of the results.

Keywords. Existence and uniqueness, Iterative method, Fractional derivatives and integrals, Fractional ordinary differential equations, Nonlinear boundary value problem.

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1 Introduction

Many studies have been conducted on iteration techniques because of their broad range of applications. When defining the iteration method, there are two major points to consider. The first is that the iteration to be defined is faster than existing iteration methods, and the second is simplicity.

Quite recently, V. Karakaya, Y. Atalan, K. Dogan, and NH. Bouzara [16] introduced the following three-steps iteration process:

$$
\begin{cases}
x_0 \in X, \\
x_{k+1} = Ty_k, \\
y_k = (1 - \xi_k)z_k + \xi_k T z_k, \\
z_k = Tx_k, \quad k \in N \cup \{0\}.
\end{cases}
\tag{1}
$$