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EXISTENCE OF PERIODIC SOLUTIONS AND BIFURCATION OF FRACTIONAL DIFFERENTIAL EQUATION

Fatemeh Keshtkar

Department of Mathematics
Jahrom University, Jahrom, Iran

Abstract. In this work, we introduce a class of ordinary differential equations (ODEs) that, under certain conditions, exhibit periodic orbits and include some well-known Linard systems. Next, we extend the study to their fractional counterparts, focusing on their stability and periodic solutions under fractional derivatives $0 < \alpha < 1$, which have an asymptotically stable equilibrium at the origin as their ω-limit set. Finally, we consider a typical ODE example that is related to this class of systems, and besides that, satisfies the conditions of the Hopf bifurcation theorem. Then we study the bifurcation of this typical example presented by fractional order derivative in two cases, first by perturbing the system by changing its parameter value and second by changing its fractional order derivative. Through this bifurcation, we conclude that the appearance of the limit cycle in a FDE system is sensitive to the value of the fractional order derivative. The numerical results are illustrated.

Keywords. Lienards systems, Heteroclinic orbit, Hopf Bifurcation,
Limit Cycle, Fractional Differential Equations.

AMS (MOS) subject classification: 37G15, 34C05, 34A08

1 Introduction

Most classical studies in dynamical systems are devoted to ordinary differential equations (ODEs). However, during the past decades it has become clear that fractional-order differential equations (FDEs) offer a more accurate description for many real-world processes, especially those involving memory and hereditary effects. Although the theory of ODEs is well developed, the corresponding theory for fractional systems is still limited, and many classical results do not carry over directly to the fractional setting.
Among the various types of dynamical systems, those that exhibit periodic orbits or limit cycles are of particular interest. Periodic solutions model phenomena with regularly repeating behavior are therefore important in applications ranging from biology and physics to economics, mechanics, and engineering. Because of their predictive nature, identifying conditions that guarantee the existence of periodic orbits is a central problem in the qualitative theory of differential equations [4, 5, 9, 15, 11, 21, 23]. The importance of