A STUDY OF QUASILINEAR ELLIPTIC OBSTACLE PROBLEMS: EMPLOYING YOUNG MEASURE THEORY AND THE KINDERLEHRER-STAMPACCHIA THEOREM

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Abstract. We investigate a class of obstacle problems in Sobolev spaces of the form

$$\omega \in \Upsilon_{\varpi,\varrho} : \int_{\wp} \Theta(|D\mathbf{u}|)D\mathbf{u}) : D(\omega - \mathbf{u}) + \sigma(\mathbf{u}) : D(\omega - \mathbf{u}) \; d\eta \geq 0,$$

and demonstrate the existence and uniqueness of a weak solution using Young measure theory along with a theorem by Kinderlehrer and Stampacchia. Given that our operator lacks the monotonicity property required for the proof, we introduce an alternative condition to address this issue.

Keywords. Obstacle problem, Theorem of Kinderlehrer and Stampacchia, Young measures.

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1 Introduction

Nonlinear elliptic obstacle problems are prevalent in various fields such as fluid mechanics, material science, and image processing. One notable example involves determining the equilibrium shape of a flexible membrane that is constrained by an obstacle. In this scenario, the membrane is modeled as a two-dimensional surface under specific tension, with an obstacle positioned upon it.

The objective is to identify the membrane's shape that minimizes the system's total energy while ensuring that it does not penetrate the obstacle. This problem has numerous applications, including the design of inflatable structures, the investigation of biological membranes, and the analysis of elastic sheets in materials science.

This paper aims to explore the existence and uniqueness of a weak solution for the corresponding obstacle problem of the form:

$$\omega \in \Upsilon_{\varpi,\varrho} : \int_{\wp} \Theta(|D\mathbf{u}|)D\mathbf{u}) : D(\omega - \mathbf{u}) + \sigma(\mathbf{u}) : D(\omega - \mathbf{u}) \; d\eta \geq 0 \quad (1.1)$$