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APPROXIMATION RESULTS FOR LOCAL SOLUTION OF INITIAL VALUE PROBLEMS OF NONLINEAR FIRST ORDER ORDINARY HYBRID FUNCTIONAL INTEGRODIFFERENTIAL EQUATIONS

Janhavi B. Dhage, Shyam B. Dhage and Bapurao C. Dhage

Kasubai, Gurukul Colony, Thodga Road
Ahmedpur - 413515, Dist. Latur
Maharashtra, India

Abstract. In this paper, we establish a couple of approximation results for local existence and uniqueness of the solution of an IVP of nonlinear first order ordinary hybrid functional integrodifferential equations by using the Dhage monotone iteration method based on the recent hybrid fixed point theorems of Dhage (2022) and Dhage *et al.* (2022). An approximation result for Ulam-Hyers stability of the local solution of the considered hybrid differential equation is also established. Finally, our main abstract results are also illustrated with the help of a couple of numerical examples.

Keywords. Integrodifferential equation; Hybrid fixed point principle; Dhage iteration method; Approximation theorem, Ulam-Hyers stability.

AMS (MOS) subject classification: 34A12, 34A34, 34A45, 47H10.

1 Introduction

Given a closed and bounded interval $J = [t_0, t_0 + a]$ of the real line $\mathbb{R}$, for some $t_0, a \in \mathbb{R}$ with $a > 0$, we consider the initial value problem (IVP) of nonlinear first order ordinary hybrid functional integrodifferential equations (in short HFIGDEs),

$$\left.\begin{aligned} x'(t) + h(t)x(t) &= f\left(t, x(t), \int_{t_0}^{\sigma(t)} g(s, x(s))\, ds\right),\ t \in J, \\ x(t_0) &= \alpha_0 \in \mathbb{R}, \end{aligned}\right\} \tag{1}$$

where $h \in L^1(J, \mathbb{R}_+)$, $\sigma : J \to J$ is continuous with $\sigma(t) \leq t,\ t \in J$ and the functions $f : J \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ and $g : J \times \mathbb{R} \to \mathbb{R}$ satisfy certain hybrid, that is, mixed hypotheses from algebra, analysis and topology to be specified later.

Definition 1.1. A function $x \in C(J, \mathbb{R})$ is said to be a *solution* of the HFIGDE (1) if it satisfies the equations in (1) on J, where $C(J, \mathbb{R})$ is the space of continuous real-valued functions defined on J. If a solution x lies in a neighborhood $\mathscr{N}(x_0)$ of some point $x_0 \in C(J, \mathbb{R})$, then we say it is a *local solution* or *neighborhood solution* (in short *nbhd solution*) of the HFIGDE (1) defined on J.