

EXISTENCE AND STABILITY OF SOLUTIONS FOR A DELAYED FRACTIONAL-ORDER DIFFERENTIAL PROBLEM INVOLVING THE CAPUTO FABRIZIO DERIVATIVE

Lekbir Monsif¹, Abderrahman Elgmairi¹, Jalila El Ghordaf¹ and
Mohamed Oukessou¹

¹Laboratory of Applied Mathematics and Scientific Computing, Sultan Moulay Slimane
University, P.O.Box 532, Beni Mellal, 23000, Morocco

Abstract. The primary goal of this paper is to prove the existence and uniqueness of the solution of fractional differential equation involving time delay. The fundamental instrument for obtaining this result is the fixed point theorem. Secondly, the Ulam-Hyers stability of the above problem will be studied.

Keywords. Caputo-fabrizio fractional integral; Caputo-fabrizio fractional derivative; Banach fixed point theorem, Schauder's fixed point theorem, Hyers-Ulam stability.

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1 Introduction

Fractional differential equations (FDEs) have emerged as powerful mathematical tools for modeling complex phenomena with memory and hereditary properties across diverse scientific disciplines, including biomedical engineering, neuroscience, ecology, viscoelasticity, fracture mechanics, and the study of anomalous diffusion in physics. Beyond these applications, FDEs provide a rigorous framework for analyzing chaotic and nonlinear dynamical systems, offering insights that are unattainable through classical integer-order models. For a comprehensive overview, we refer the reader to the established works of Abbas et al. [22], Miller and Ross [10], and others [23, 26, 25, 1, 2, 3], along with the references therein.

In recent years, significant attention has been devoted to fractional differential equations incorporating time delays, as such delays arise naturally in biological systems, control theory, population dynamics, and communication networks. However, the presence of delays substantially complicates the analysis, introducing challenging questions regarding the existence, uniqueness, and stability of solutions. Addressing these questions is crucial for ensuring the reliability of mathematical models in real-world applications. A considerable body of work has been dedicated to exploring these equations