

## EXISTENCE RESULTS FOR HYBRID DIFFERENTIAL EQUATIONS WITH TEMPERED $\psi$ -CAPUTO FRACTIONAL OPERATORS

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**Abstract.** In this paper, we investigate the existence of solutions for nonlinear hybrid fractional differential equations involving the tempered  $\psi$ -Caputo fractional derivative of order  $\gamma \in (0, 1)$ . The existence theorem is proved using the Dhage fixed point theorem and some basic tools of  $\psi$ -tempered fractional calculus. Finally, an illustrative example is given to verify our theoretical results.

**Keywords.** Hybrid differential equation, Tempered  $\psi$ -fractional integral, Tempered  $\psi$ -Caputo fractional derivative, Carathodory function, Existence theorem.

**AMS (MOS) subject classification:** 34A08, 34K37.

## 1 Introduction

Fractional calculus extends the principles of regular differentiation and integration to non-integer orders. In recent years, substantial progress has been made in the field of fractional processes. These processes play a crucial role in characterizing the long-memory behavior of diverse time series. Physicists are especially driven to investigate fractional differential equations, recognizing their significance in physics and related domains. This leads to the development of fractional calculus, which encompasses various types of fractional derivatives, such as Riemann-Liouville [17], Hilfer [16], Erdelyi-Kober [20] and Hadamard [1]. The motivation for introducing a new fractional derivative is twofold: first, to capture some dynamics of physical systems that are not well represented by existing fractional derivatives, and second, to preserve the characteristics of the standard derivative. A remarkable example of these derivatives is the tempered fractional derivative, which is especially useful for modeling biological processes that involve the movement of particles in confined spaces with limited time see [6, 11, 19].

Recently, Fahad et al. [13] introduced the tempered  $\psi$ -Caputo derivative, which is a tempered fractional derivative with respect to a function in the sense Caputo. This notion fractional calculus with respect to a function, has