Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **32 (2025)** 325-338
 http://www.watam.org

BLOW-UP PHENOMENA FOR PSEUDO-PARABOLIC EQUATIONS WITH LOGARITHMIC NONLINEARITY OF VARIABLE EXPONENTS

Abdelatif Toualbia

Department of Mathematics
Echahid Cheikh Larbi Tebessi University, Tebessa, Algeria

Abstract. We consider pseudo-parabolic equations with variable exponents and logarithmic nonlinear term subject to Dirichlet boundary conditions

$$\upsilon_t - \Delta\upsilon_t - div(A(x,t)\left|\nabla\upsilon\right|^{r(x)-2}\nabla\upsilon) = \left|\upsilon\right|^{s(x)-2}\upsilon\ln(\left|\upsilon\right|).$$

Using a differential inequality technique, we prove that the solutions become unbounded at a finite time T, and find an upper bound for this time with negative initial energy. Also, a lower bound for blow-up time is determined.

Keywords. Pseudo-parabolic equation; Blow-up; Upper bound; Lower bound; Variable exponents.

AMS (MOS) subject classification: 35K70, 35B44, 34L15.

1 Introduction

Let Ω represent a bounded domain in $\mathbb{R}^n (n \geq 1)$ with smooth boundary $\partial\Omega$, and we consider the following pseudo-parabolic equation

$$\begin{cases} \upsilon_t - \Delta\upsilon_t - div(A(x,t)\left|\nabla\upsilon\right|^{r(x)-2}\nabla\upsilon) = \left|\upsilon\right|^{s(x)-2}\upsilon\ln\left|\upsilon\right|, & \text{in} Q_T, \\ \upsilon(x,t) = 0, & \text{on } \partial Q_T, \\ \upsilon(x,0) = \upsilon_0(x), & x \in \Omega, \end{cases} \tag{1}$$

where $Q_T = \Omega \times (0,T)$, $\partial Q_T = \partial\Omega \times (0,T)$. $(.)'$ denotes the derivative with respect to time t thus $\upsilon_t = \frac{\partial\upsilon}{\partial t}$, $div(A(x,t)\left|\nabla\upsilon\right|^{r(x)-2}\nabla\upsilon)$ is the so-called $r(x)$-Laplace operator with the presence of a matrix $A(x,t)$. The logharitmic nonlinearity $\left|\upsilon\right|^{s(x)-2}\upsilon\ln(\left|\upsilon\right|)$ plays the role of a source, and the dissipative term $\Delta\upsilon_t$ is a linear strong damping term.
The matrix $A = \left(a_{ij}(x,t)\right)_{i,j}$ where a_{ij} is a function of class $C^1(\overline{\Omega} \times [0,\infty[)$ and there exists a constant a_0 such that, for all $(x,t) \in \overline{\Omega} \times [0,\infty[$ and $\xi \in \mathbb{R}^n$, we have

$$A\xi.\xi \geq a_0\left|\xi\right|^2 \tag{2}$$