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NONLINEAR FRACTIONAL Q-DIFFERENTIAL EQUATIONS INVOLVING HILFER-KATUGAMPOLA DERIVATIVES OF MOVING ORDERS

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Abstract. This study comprehensively investigates the existence, uniqueness, and stability of solutions for nonlinear fractional q-differential equations involving Hilfer-Katugampola q-derivatives of moving orders. We apply the Banach contraction principle and Schauder's fixed-point theorem to establish the existence of solutions. Furthermore, we examine the stability of the solutions using Ulam-Hyers theorems. Two detailed examples are provided to illustrate the practical applicability and validity of our theoretical results.

Keywords. q-calculus; q-Hilfer-Katugampola fractional derivative; existence; uniqueness; Ulam-Hyers stability.

AMS (MOS) subject classification: 05A30; 26A33; 33D05; 34K37; 39A13.

1 Introduction

Fractional calculus, which extends the classical concepts of differentiation and integration to non-integer orders, has garnered significant attention due to its powerful ability to model memory and hereditary properties in complex systems. It has found widespread applications in various scientific and engineering domains, including blood flow dynamics, electrical circuits, biology, chemistry, physics, control theory, wave propagation, and signal and image processing. For comprehensive insights into its practical applications, readers are referred to the works of Afshari et al. [1, 2, 3], Agrawal [4], Basti *et al.* [5, 6, 7, 8], Benchohra *et al.* [9, 10, 11, 12, 13], Herrmann [14], Hilfer [15], and Kilbas *et al.* [16].

In parallel, the twentieth century witnessed a revolutionary development in quantum mechanics, which inspired the emergence of quantum calculus, a framework introduced by Jackson in 1909 ([17]). This branch of calculus, which avoids the traditional concept of limits, is deeply influential in mathematics, mechanics, and physics [18, 19, 20]. Recognizing its potential, researchers such as Al-Salam and Agarwal extended the theory to fractional q-calculus, a synthesis of fractional and quantum calculus, to better model physical, biological, and economic systems [21, 22].