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FIRST-ORDER NONLINEAR INTEGRO-DIFFERENTIAL EQUATIONS WITH NONLOCAL CONDITION

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Abstract. In this paper, the existence of solution of a first-order nonlinear integro-differential equation with finite and infinite boundary conditions is studied by using Schauder's fixed point theorem. The uniqueness of that solution, continuous dependence of the solution on initial condition φ_0 and on the nonlocal data a_k have studied. An example is provided to illustrate the main results.

Keywords. Functional equations, existence of solutions, uniqueness of solution, continuous dependence, nonlocal conditions.

1 Introduction

Lately, the researchers have interested to study many important nonlocal boundary value problems which gives importance. Also, many papers were discussed the existence solutions of various shapes of functional differential equation boundary value problems, for more details, the reader is referred to [2, 3, 5–10, 10, 12–15, 17–25] and the references therein.
In this paper, we investigate the existence and uniqueness of a functional of the integro-differential equation with nonlocal condition which have the following form:

$$\begin{cases} \frac{d\varphi(\zeta)}{d\zeta} = f\left(\zeta, \varphi(\zeta), \varphi'(\zeta), \int_0^\zeta g\left(\eta, \varphi(\eta), \varphi'(\eta)\right) d\eta\right), \quad a.e. \quad \zeta \in (0,1], \\ \sum_{k=1}^m a_k \varphi(\tau_k) = \varphi_0, \quad a_k \geq 0 \quad \tau_k \in (0,1). \end{cases} \tag{1.1}$$

The continuous dependence of the solution on the initial condition φ_0 and on the nonlocal data a_k, has been explored.
As application, we investigate the nonlocal boundary value problem of functional integro-differential equation with the infinite-point boundary condition

$$\begin{cases} \frac{d\varphi(\zeta)}{d\zeta} = f\left(\zeta, \varphi(\zeta), \varphi'(\zeta), \int_0^\zeta g\left(\eta, \varphi(\eta), \varphi'(\eta)\right) d\eta\right), \quad a.e. \quad \zeta \in (0,1], \\ \sum_{k=1}^\infty a_k \varphi(\tau_k) = \varphi_0, \quad a_k \geq 0 \quad \tau_k \in (0,1), \text{if} \sum_{k=1}^\infty a_k \text{ is convergent}, \end{cases} \tag{1.2}$$