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THE UNSTABLE MANIFOLD HYPOTHESIS

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Abstract. The combination of the impossibility of long-term prediction, the short-term nature of human thought and the increasing complexity of civilization implies the need to extend the analysis of hyperbolic dynamical systems to the subject of evolving hyperbolic dynamical systems and its attendant unstable manifolds.
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1 Introduction: A short history of the unstable manifold:

The complexity of the unstable manifold was described in the 1890s by Poincar'e[1] and Fig. 1, Plate A. In the 1980s Smale ,in the Smale Birkhoff theorem [2], linked the complexity described by Poincar'e to the shift, a metaphor for a coin toss [3]. The connection to the shift linked the complexity of the unstable manifold to the emergence of the concept of chaos [4]. Until the 1990s, the complexity of the unstable manifold was represented by drawings of the homoclinic tangles of the stable and unstable manifolds.

The next step needed was suggested by by Hirsch [5] that the presence of chaos should be clear from the *form* of an ODE. This would provide a means of more quickly recognizing when chaos was present in the solution of the ODE. In order to link the form of an ODE to chaos required the development of an ODE form from which chaos could be recognized. This was the IDE form [8]. The significance of the IDE form was two fold: (a) it provided techniques for recognizing when the elements that that produced chaos were present in an ODE; and, (b) in turn, the ability to recognize when the elements of chaos were present in the form of an ODE also demonstrated how to derive chaotic systems *wholesale*.

As a result of (b) above, far greater detail about how varied unstable manifolds can be emerged. The variation of the geometry of unstable manifolds displaced the conventional hand drawings of homoclinic tangles.

Given an autonomous ODE with a parameter,a, by slowly increasing the parameter the solution of the ODE can pass in and out of chaotic dynamics