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TIME SCALE DISTINGUISHABILITY OF LINEAR IMPULSIVE CONTROL SYSTEMS

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Abstract. When two systems are distinguishable, their output trajectories cannot be identical regardless of their initial states and control inputs that are applied to them. In this paper, we analyze the time scale distinguishability of two distinct linear piecewise impulsive control dynamical systems to establish equivalent criteria for such distinguishability. Furthermore, a piecewise impulsive transition matrix is established along with some of its characteristics. Some examples explaining the idea of time scale distinguishability are also discussed.

Keywords. impulsive control system; distinguishability; observability; transition matrix; time scale.

AMS subject classification: 15A22, 34A30, 93B07.

1 Introduction

Impulsive differential equations are used as a mathematical apparatus for analyzing the behavior of most real-life processes. The mathematical tool for designing impulsive control systems is the theory of these equations, which expresses processes that change their states at certain non-fixed or fixed moments in the form of 'jumps', see [2, 3]. The impulsive control theory is used in many fields such as control and synchronization, chaos, chemical systems, genetic networks, neural networks, etc., see e.g. [22, 23]. Impulsive control differential equations have also become increasingly important in physical sciences, e.g. in population dynamics and economical theories. Recently, semi-linear second order impulsive differential equations have also been implemented in control theory, see [35]. There used to be a gap where no theory could address both the continuous and discrete cases simultaneously. Hilger first proposed the time scales theory, which can unify discrete and continuous analysis, see [18, 19]. The work of Bohner and Peterson [8, 9] on dynamic equations and their advancements is among the most contributed work on time scales.

On the other hand, observability and distinguishability are interconnected concepts in control theory. In the context of linear systems, observability de-