

## ON SOME NEW DYNAMIC INEQUALITIES OF HARDY-HILBERT-TYPE ON TIME SCALES

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**Abstract.** This paper is devoted to the investigation of Hardy-Hilbert inequality on time scales. Our methods rely on the extended, as special cases, the continuous and discrete inequalities. These inequalities generalized and extended existing results in the literature. We use Hölder's inequality on time scales and a few algebraic inequality to demonstrate our conclusions.

**Keywords.** Hardy-Hilbert's inequality, dynamic inequality, time scales, nabla calculus.

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## 1 Introduction

The discrete version of Hardy-Hilbert inequalities is stated as [1]:

$$\sum_{s=1}^{\infty} \sum_{\iota=1}^{\infty} \frac{\phi_s \psi_{\iota}}{(s+\iota)^{\lambda}} \leq K \left( \sum_{s=1}^{\infty} \phi_s^{\nu} \right)^{\frac{1}{\nu}} \sum_{\iota=1}^{\infty} \left( \psi_{\iota}^{\varpi} \right)^{\frac{1}{\varpi}},$$

You may get the integral analogue of the above inequality in [1].

$$\int_0^{\infty} \int_0^{\infty} \frac{\phi(\iota)\psi(\varsigma)}{(\iota+\varsigma)^{\lambda}} d\iota d\varsigma \leq K \left( \int_0^{\infty} \phi^{\nu}(\iota) d\iota \right)^{\frac{1}{\nu}} \left( \int_0^{\infty} \psi^{\varpi}(\varsigma) d\varsigma \right)^{\frac{1}{\varpi}},$$

where  $K = K(\nu, \varpi)$  depends on  $\nu$  and  $\varpi$  only.