

GLOBAL EXISTENCE FOR A RESCALED BOUSSINESQ SYSTEM IN CRITICAL FOURIER-BESOV-MORREY SPACES

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Abstract. In this paper, we study a rescaled Boussinesq system in \mathbb{R}^3 . By using the Fourier localization argument and the Littlewood Paley theory, we establish the global well-posedness result in critical Fourier-Besov-Morrey spaces for small initial data.

Keywords. Fourier-Besov-Morrey spaces, global existence, rescaled Boussinesq system, Fourier localization argument, Littlewood Paley theory.

AMS (MOS) subject classification: 35Q35, 35k55.

1 Introduction

In this study, we focus on a specific case of the following family of Boussinesq equations in n dimensions,

$$\begin{cases} \partial_t u - \mu \Delta u + (u \cdot \nabla) u + \nabla \pi = b e_n & (t, x) \in \mathbb{R}^+ \times \mathbb{R}^n, \\ \partial_t b - \eta \Delta b + (u \cdot \nabla) b = 0 & (t, x) \in \mathbb{R}^+ \times \mathbb{R}^n, \\ \nabla \cdot u = 0 \\ u|_{t=0} = u_0, \quad b|_{t=0} = b_0 & x \in \mathbb{R}^n, \end{cases} \quad (1)$$

in this context, the unknown functions are represented by the n -dimensional velocity vector field $u(t, x) = (u^1(t, x), u^2(t, x), \dots, u^n(t, x))$. The scalar quantity $b = b(t, x)$ signifies various physical properties, such as temperature variation in a gravity field, particularly relevant in scenarios involving thermal convection. The scalar function $\pi = \pi(t, x)$ corresponds to pressure. The parameters, μ and η , denote the viscosity and thermal diffusivity, respectively.

The Boussinesq equations are a set of mathematical equations that elucidate the intricate interplay between thermal convection and fluid dynamics. They constitute a foundational model essential for understanding various geophysical phenomena, such as ocean circulation, atmospheric fronts, and scenarios characterized by the influential factors of rotation and stratification