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K-TYPE CHAOS OF $\mathbb{Z}^D$-ACTIONS

Anshid Aboobacker and Sharan Gopal

Department of Mathematics, BITS Pilani, Hyderabad Campus, India

Abstract. In this paper, we define and study the notions of $k-$type proximal pairs, $k-$type asymptotic pairs and $k-$type Li-Yorke sensitivity for dynamical systems given by $\mathbb{Z}^d$-actions on compact metric spaces. We prove the Auslander-Yorke dichotomy theorem for $k-$type notions. The preservation of some of these notions under uniform conjugacy is also studied. We also study relations between these notions and their analogous notions in the usual dynamical systems.

Keywords. Chaos, $k-$type chaos, Li-Yorke Sensitivity, induced actions, Auslander-Yorke dichotomy theorem.

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1 Introduction

A pair (X, f) where X is a compact metric space and f is a homeomorphism on X is called a *dynamical system*. The dynamics on X is given by the group $\{f^n : n \in \mathbb{Z}\}$ where $f^n = f \circ f \circ \cdots \circ f$ (n times), $f^{-n} = f^{-1} \circ f^{-1} \circ \cdots \circ f^{-1}$ (n times) and f^0 is the identity function. This dynamical system can also be seen as a $\mathbb{Z}$-action on X, where $(n, x) \mapsto f^n(x)$ for every $x \in X$ and $n \in \mathbb{Z}$. We can generalize this concept to study any group action on X and in particular, the actions of $\mathbb{Z}^d$ on X. Hereafter, we write a dynamical system as (X, T) where X is a compact metric space and T is a $\mathbb{Z}^d$-action on X; the image of (n, x) under T is denoted as $T^n(x)$ for every $x \in X$ and $n \in \mathbb{Z}^d$.

In a 2007 paper [6], to study about chain recurrences in multidimensional discrete time dynamical systems, Oprocha introduced $k-$type limit sets, $k-$type limit prolongation sets and $k-$type transitivity for $\mathbb{Z}^d$-actions where k is an integer between 1 to 2^d. Following this idea, Shah and Das [7, 8] defined $k-$type sensitivity, $k-$type periodic points, $k-$type Devaney chaos, $k-$type Li-Yorke pairs, etc. They studied about preservation of many of these notions under conjugacy and uniform conjugacy. Shah and Das [7] also looked into relationship between these notions in $\mathbb{Z}^d$-actions and their induced actions on hyperspace $K(X)$, the space of all compact subsets of X.

Kamarudin and Dzul-Kifli [4] considered a dynamical system (X, f) and studied the induced $\mathbb{Z}^d$-actions on X. They showed that transitivity in the base system persists in the induced system under certain conditions.