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SOME REMARKS ON ANOSOV FAMILIES

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Abstract. We study Anosov families which are sequences of diffeomorphisms along compact Riemannian manifolds such that the tangent bundle split into expanding and contracting subspaces. In this paper we verify that a certain class of Anosov families: (i) admit canonical coordinates, (ii) are expansive, (iii) satisfy the shadowing property, and (iv) exhibit a Markov partition.

Keywords. Anosov families, Anosov diffeomorphisms, Markov partitions, uniform hyperbolicity, non-autonomous dynamical systems, expansiveness, shadowing.

AMS (MOS) subject classification: 37C60, 37D20.

1 Introduction

An Anosov family is a (biinfinite) sequence of diffeomorphisms defined on a sequence of compact Riemannian manifolds, with an invariant sequence of splittings of the tangent bundle into expanding and contracting subspaces, and with uniform upper bound for the contraction and lower bound for the expansion.

Anosov families (Definition 2.3) were introduced by P. Arnoux and A. Fisher in [7], motivated by the generalization of the notion of Anosov diffeomorphisms. The authors focused their studies on linear Anosov families on the two-torus, aiming to develop a natural notion of completion for the set of all orientation-preserving linear Anosov diffeomorphisms on the two-torus (see [7]). Young [27] demonstrated that families consisting of C^{1+1} perturbations of an Anosov diffeomorphism of class C^2 are Anosov families. In [15] and [24], the authors investigated entropy formulas for non-stationary subshifts of finite type. Muentes, in [17, 18, 19], studied structural stability and the stable and unstable manifold theorems for Anosov families on compact Riemannian manifolds. In this work, we will explore certain properties related to hyperbolicity for Anosov families.