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EXPLORING BIFURCATION AND OPTIMAL CONTROL DYNAMICS IN CAPUTO FRACTIONAL SOIL-PLANT INTERACTION SYSTEMS

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Abstract. This study highlights the significance of employing a non-integer Caputo fractal fractional-order approach to model soil pollution through a three-compartment framework, encompassing soil pollution, metabolic activity, and plant growth. The analysis involves identifying equilibrium points, including steady-state and boundary conditions. Key dynamic shifts are explored using stability and bifurcation analysis. Optimal control strategies are developed based on Pontryagin's Maximum Principle, aiming to mitigate soil pollution and enhance plant growth. An approximate solution is obtained using Adam's Bashfoth predictor corrector method, with graphical validations conducted via MATLAB.

Keywords. Eco-Epidemiology, Stability, Bifurcation, Optimal Control, Numerical Approximation.

AMS (MOS) subject classification: Primary: 54H25, Secondary: 47H10.

1 Introduction

This study addresses the complex issue of environmental pollution, focusing on external factors such as heavy metalslike mercury and leadthat pose significant hazards to both living and non-living organisms. These toxic substances result in the degradation of soil, which adversely affects plant growth by depleting essential nutrients [2], [6]. Industrial waste, pesticides, herbicides, insecticides, and fungicides are identified as primary contributors to soil pollution. Soil degradation is categorized into two types: natural processes and anthropogenic activities. While several researchers have explored the controllability of pollution, most studies have primarily focused on water, air, and atmospheric pollution, leaving soil pollution relatively underexplored [7], [11]. In this research, the nonlinear impact of soil pollution on plant growth is analyzed. Mathematical modeling, a powerful tool for understanding dynamic systems, is employed to study the behavior of plant-soil interactions under pollution. These models have wide-ranging applications in fields such as population dynamics, ecological evolution, infectious disease spread, engineering systems, and neurobiology [12], [15]. In recent years, fractional differential equations (FDEs) have gained attention due to their ability to model complex systems with memory effects [1],[8]. These equations find