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EXISTENCE OF MULTIPLE SOLUTIONS TO A FRACTIONAL BOUNDARY VALUE PROBLEM WITH A P-LAPLACIAN AND IMPULSIVE EFFECTS

Shahin Moradi[1] Ghasem A. Afrouzi[1] and John R. Graef[2]

[1]Department of Mathematics
Faculty of Mathematical Sciences
University of Mazandaran, Babolsar, Iran

[2]Department of Mathematics
University of Tennessee at Chattanooga
Chattanooga,TN 37403 USA

Abstract. Sufficient conditions for the existence of multiple classical solutions to a fractional boundary value problems with impulses are established. Critical point theory is the main technique used in the proofs. An example is presented to illustrate the primary results.

Keywords. Fractional differential equations; Impulsive effects; Infinitely many solutions; Variational methods; p-Laplacian.

AMS (MOS) subject classification: 35A09, 35R12, 34A08.

1 Introduction

Our purpose here is to investigate the existence of multiple classical solutions to the nonlinear fractional boundary value problem with impulses (BVP)

$$\begin{cases} D^{\alpha}_{-T}\Phi_p({}^cD^{\alpha}_{0^+}u(t)) + |u(t)|^{p-2}u(t) = \lambda f(t,u(t)), & t \neq t_j, \quad t \in (0,T), \\ \Delta(D^{\alpha-1}_{-T}\Phi_p({}^cD^{\alpha}_{0^+}u))(t_j) = I_j(u(t_j)), & j = 1,\dots,m, \\ u(0) = u(T) = 0, \end{cases} \tag{P^f_λ}$$

where $\alpha \in (\frac{1}{p}, 1]$, $p > 1$, and $\Phi_p(s) = |s|^{p-2}s$ $(s \neq 0)$. Here, D^{α}_{-T} is the right-hand Riemann-Liouville fractional derivative of order α, and ${}^cD^{\alpha}_{0^+}$ is the left-hand Caputo fractional derivative of order α. Also, for each $j = 1,\dots,m$,

$$\Delta(D^{\alpha-1}_{-T}\Phi_p({}^cD^{\alpha}_{0^+}u))(t_j) = D^{\alpha-1}_{-T}\Phi_p({}^cD^{\alpha}_{0^+}u)(t_j^+) - D^{\alpha-1}_{-T}\Phi_p({}^cD^{\alpha}_{0^+}u)(t_j^-),$$

$$D^{\alpha-1}_{-T}\Phi_p({}^cD^{\alpha}_{0^+}u)(t_j^+) = \lim_{t\to t_j^+} D^{\alpha-1}_{-T}\Phi_p({}^cD^{\alpha}_{0^+}u)(t),$$

$$D^{\alpha-1}_{-T}\Phi_p({}^cD^{\alpha}_{0^+}u)(t_j^-) = \lim_{t\to t_j^-} D^{\alpha-1}_{-T}\Phi_p({}^cD^{\alpha}_{0^+}u)(t),$$