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THE HIRSCH CONJECTURE REVIEWED

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Abstract. This paper reviews the progress made in resolving the Hirsch conjecture.

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1 Introduction

> *"A major challenge to mathematicians is to determine which dynamical systems are chaotic and which are not. Ideally one should be able to tell from the form of the differential equation"*
> – Morris W. Hirsch, 1985 [1], page 192.

Three issues arise that must be addressed: 1) There does not exist a rigorous universally accepted operational definition of chaos. By an *operational definition* is meant that scientist, researchers and engineers can use the definition to efficiently determine if a system of interest can be determined to be chaotic, or equivalently, unpredictable and to what degree. 2) Every accepted example of chaos has initial conditions that do not produce chaos. 3) There exist only a sparse set of generally accepted examples of chaos. The proof of theorems relies to a great extent on the existence of many useful examples and counterexamples that can be used to guide theorem derivation. .

As to issue 1, it may be that the existence of a chaotic sequence is equivalent to the Axiom of Choice. As to issue 2, there cannot be a pure chaotic or random sequence. Issue 3 is addressed by IDE Theory.

Before IDE Theory [2], there were only a sparse set of chaotic systems.. Hence, the first problem of Hirsch Conjecture was how to increase the example space of chaotic dynamical systems to be used for analysis, , see Sec. 5, 6, 7. The answer to that question was the development of IDE theory see, Sec 2 and a subset of components occurring in chaotic systems is found in Sec. 4.

The first time series that needed derivation was a time series for which the unilateral shift was the time one map, Sec. 3. Selecting the shift time series to be constructed