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FINITE ELEMENT METHOD TO SOLVE CAUCHY PROBLEMS FOR DEGENERATE HYPERBOLIC EQUATIONS

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Abstract. The primary aim of this article is to effectively utilize the finite element method to intricately deduce a numerical solution for a second-order degenerate hyperbolic partial differential equation in terms of time. Furthermore, within this paper, our overarching intention is to thoroughly explore and analyze a novel approach based on reducing second-order time derivatives to first order, offering a well-founded solution strategy tailored to these types of hyperbolic equations. Through this comprehensive investigation, we strive to contribute to a deeper understanding and potential advancement of methodologies for solving such complex mathematical equations.

Keywords. Degenerate Partial Differential Equations, Transmutation Methods, Kolmogorov Equation, Inverse Laplace Transform, Finite Element Method, Numerical Analysis.

AMS (MOS) subject classification: 65M60, 35L65, and 35Q79.

1 Introduction

The primary aim of this article is to effectively utilize the finite element method to intricately deduce a numerical solution for the following second-order degenerate hyperbolic partial differential equation (PDE) in terms of time. Additionally, we explore an explicit transmutation-based solution to this PDE.

$$\begin{cases} \partial_{tt}u - \Delta_x u - \langle x, \nabla_y u\rangle = 0\ , & \mathbb{R}^n_x \times \mathbb{R}^n_y \times (0,\infty), \\ u(x,y,0) = 0\ , & \partial_t u(x,y,0) = \psi(x,y). \end{cases} \tag{1.1}$$

This PDE is highly degenerate because it lacks the diffusive term $\Delta_y u$.The corresponding parabolic Cauchy problem was first introduced by Andrey Kolmogorov in a famous 1934 note [2].