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CAUCHY PROBLEMS FOR HILFER FRACTIONAL IMPULSIVE EVOLUTION EQUATIONS ON AN INFINITE INTERVAL

K. Karthikeyan[1] and G.S. Murugapandian[2]

[1]Department of Mathematics, KPR Institute of Engineering and Technology, Coimbatore-641407, Tamil Nadu, India. karthi_phd2010@yahoo.co.in
[2]Department of Mathematics, Nandha Engineering College, Erode-638052, Tamil Nadu, India. murugapandian.g.s@gmail.com

Abstract. It is well known that the classical Ascoli-Arzel theorem is powerful technique to give a necessary and sufficient condition for investigating the relative compactness of a family of abstract continuous functions, while it is limited to finite compact interval. In this paper, we shall generalize the Ascoli-Arzel theorem on an infinite interval. As its application, we investigate an initial value problem for impulsive fractional evolution equations on infinite interval in the sense of Hilfer type, which is a generalization of both Riemann-Liuoville and Caputo fractional derivatives. Our methods are based on the Hausdorff theorem, classical generalized Ascoli-Arzel theorem, Schauder fixed point theorem,Wright function, and Kuratowski measure of noncompactness.We obtain the existence of mild solutions on an infinite interval when the semigroup is compact as well as noncompact.
KEYWORS: Ascoli-Arzel theorem, existence, impulsive fractional evolution equations, Hilfer derivative, infinite interval
MSC CLASSIFICATION: 26A33, 34A08, 34K37

1 Introduction

Consider an initial value problem of impulsive fractional evolution equations on infinite interval

$$\begin{cases} ({}^H\mathfrak{D}_{0+}^{\rho,\zeta}\mathfrak{Y})(\mathsf{t}) = \mathscr{A}\mathfrak{Y}(\mathsf{t}) + \mathscr{G}(\mathsf{t},\mathfrak{Y}(\mathsf{t})), \quad \mathsf{t} \in (0,\infty). \\ \Delta\mathfrak{Y}|_{\mathsf{t}=\mathsf{t}_k} = \mathscr{I}_k(\mathfrak{Y}(\mathsf{t}_k^-)), k = 1,2,3,...m, \\ (\mathscr{I}_{0+}^{(1-\zeta)(1-\rho)}\mathfrak{Y})(0) = \wp_0 \in \mathbb{X}, \end{cases} \tag{1.1}$$

where ${}^H\mathfrak{D}_{0+}^{\rho,\zeta}$ is the Hilfer fractional derivative of order $\zeta \in (0,1)$ and of type $\rho \in [0,1]$, Riemann-Liouville integral $I_{0+}^{(1-\zeta)(1-\rho)}$ of order $(1-\zeta)(1-\rho)$, operator A denotes the infinitesimal generator of a strongly continuous semigroup of bounded linear operators $\{T(\mathsf{t})\}_{\mathsf{t}\geq 0}$ in Banach space $\mathbb{X}, \mathscr{G} : [0,\infty) \times \mathbb{X} \to \mathbb{X}$ is a continuous function, $\wp_0$ is an element of $\mathbb{X}$. $\Delta\mathfrak{Y}|_{\mathsf{t}=\mathsf{t}_k}$