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ABSOLUTELY CONTINUOUS INVARIANT MEASURES FOR PIECEWISE CONVEX MAPS WITH COUNTABLE NUMBER OF BRANCHES

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We consider two classes $\mathcal{T}_{pc}^{\infty}(I), \mathcal{T}_{pc}^{\infty,0}(I)$ of piecewise convex maps $\tau : I = [0,1] \to [0,1]$ with countable number of branches. For the first class $\mathcal{T}_{pc}^{\infty}(I)$, we consider piecewise convex maps $\tau : I = [0,1] \to [0,1]$ with countable number of branches and arbitrary countable number of limit points of partition points separated from 0. For the second class $\mathcal{T}_{pc}^{\infty,0}(I)$, we consider piecewise convex maps $\tau : I = [0,1] \to [0,1]$ with countable number of branches and we assume: for any arbitrary integer $n \geq 0$, there exists a countable partition $0 = a_0 < \cdots < a_{0,-n} < a_{0,-(n-1)} < \cdots < a_{0,-2} < a_{0,-1} = a_1, a_2, a_3, \ldots, a_n, \ldots\}$ of $I = [0,1]$ with $\lim_{n\to\infty} a_{0,-n} = 0$. In this paper, we study absolutely continuous invariant measures of $\tau \in \mathcal{T}_{pc}^{\infty}(I)$ and $\tau \in \mathcal{T}_{pc}^{\infty,0}(I)$ respectively. We also consider non-autonomous dynamical systems of maps in $\mathcal{T}_{pc}^{\infty}(I)$ or $\mathcal{T}_{pc}^{\infty,0}(I)$ and prove the existence of acims for the limit map. Moreover, we prove exactness of $\tau \in \mathcal{T}_{pc}^{\infty}(I)$ and $\tau \in \mathcal{T}_{pc}^{\infty,0}(I)$ respectively.

Keywords. Absolutely continuous invariant measures; Piecewise convex maps; Countable branches; Frobenius-Perron operator; Non-autonomous dynamical systems; Exactness.

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1 Introduction

The existence and properties of absolutely continuous invariant measures (acims) of deterministic dynamical systems reflect their long time behaviour and play an important role in understanding their chaotic nature [1, 11, 12]. Let $\mathcal{B}$ a Borel σ-algebra of subsets of $I = [0,1]$ and λ be the normalized Lebesgue measure on I. Let $\{0 = b_0, b_1, b_2, \ldots\}$ be a countable partition of I and $\tau : I \to I$ be a non-singular measurable transformation. A measure μ on $\mathcal{B}$ is τ-invariant or τ preserve μ if $\mu(\tau^{-1}(A)) = \mu(A)$ for all $A \in \mathcal{B}$. The Frobenius-Perron operator $P_\tau : L^1(I, \mathcal{B}, \lambda) \to L^1(I, \mathcal{B}, \lambda)$ of τ plays an important role for the existence, approximations and properties of acims.