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FRACTIONAL DIFFERENTIAL INCLUSIONS WITH MULTI-POINT BOUNDARY CONDITIONS INVOLVING HILFER-HADAMARD DERIVATIVE

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Abstract. In this paper, we study the existence and uniqueness results of solutions for boundary value problems for Hilfer-Hadamard type fractional differential inclusions with multi-point boundary conditions. In the first part we deal with a non-convex valued right hand side of the equation and in the second part we consider the Carathéodory case, where we study also the compactness of solution sets. Finally in the last section, we conclude the paper by giving a concrete example where our main result can be applied.

Keywords. Compactness of solution set, Covitz and Nadler contraction, differential inclusions, existence solutions, fixed point theorem, mixed Hilfer-Hadamard fractional derivative, topological structure.

AMS (MOS) subject classification : Primary 26A33, 34A60, 34B15; Secondary 47H10, 47H09.

1 Introduction

Fractional differential equations have proven to be the appropriate models for various areas of engineering, physics, bio-engineering and other applied sciences. In the article [27] the authors present systems of Navier-Stokes equations on Cantor sets, which are described by the local fractional vector calculus. It is shown that the results for Navier-Stokes equations in a fractal bounded domain are efficient and accurate for describing fluid flow in fractal media. In the article [23] the authors extended the time fractional $(2+1)$-dimensional Zakharov-Kuznetsov $(Z-K)$ equation in quantum magneto-plasmas,

$$D_{\alpha}^{t}u + au\frac{\partial u}{\partial x} + b(\frac{\partial^3 u}{\partial x^3} + \frac{\partial^3 u}{\partial y^3}) + c(\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x^2}\frac{\partial u}{\partial y}) = 0.$$

They studied the symmetry of this equation, the approach is based on group analysis combined with the notion of Riemann-Liouville $(R-L)$ fractional