

PERIODIC ORBITS OF CONTINUOUS-DISCONTINUOUS PIECEWISE DIFFERENTIAL SYSTEMS WITH FOUR PIECES SEPARATED BY THE CURVE $XY = 0$ AND FORMED BY LINEAR HAMILTONIAN SYSTEMS

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Abstract. In recent years there has been a significant interest in studying the piecewise differential systems, mainly due to their wide range of applications in modeling natural phenomena. Understanding the dynamics of such systems in the plane is a significant challenge, particularly when we want to study their periodic orbits and, more specifically, their limit cycles. Consequently, numerous studies have been dedicated to investigating periodic orbits' existence or non-existence within continuous and discontinuous piecewise differential systems. However, to the best of our knowledge, this paper is one of the pioneering works analyzing the periodic orbits within a specific class of piecewise differential systems, the ones exhibiting continuity in one part of the separation line while being discontinuous in the other part.

Our study analyzes the periodic orbits of the piecewise differential systems formed by four pieces, having the curve $xy = 0$ as the separation line. In each piece, there is an arbitrary linear Hamiltonian system. Moreover, we assume that these piecewise differential systems exhibit continuity along the x -axis while discontinuous along the y -axis

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1 Introduction

A dynamical system is any system that changes over time, and ODEs provide a concise and elegant way to capture this behavior. A dynamical system can be defined as a function that describes the time evolution of a point in an ambient space, such as a parametric curve. Examples of dynamical systems that can be modeled with ODEs include the oscillation of a clock pendulum, the flow of water through a pipe, the motion of particles in the air, and