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BLOW-UP FOR SEMILINEAR WAVE EQUATIONS WITH LOGARITHMIC SOURCE TERM AT SUPERCRITICAL INITIAL ENERGY LEVEL

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Abstract. In this paper, we consider a class of semilinear wave equations with strongly damped term

$$u_{tt} - \Delta u - \Delta u_t = |u|^{p-2} u \ln |u|,$$

associated with initial and Dirichlet boundary conditions. Under certain conditions, we show that any solution with supercritical initial energy, blows up in finite time. Furthermore, an upper and a lower bounds for the blow-up time are obtained.

Keywords. Semilinear wave equation, strong damping, logarithmic nonlinearity, finite time blow-up, lower bound.

AMS (MOS) subject classification: 35B44, 35L05, 35L20.

1 Introduction

In this paper, we would like to study the blow-up of solutions of the following initial boundary value problem of a semilinear wave equation

$$\begin{cases} u_{tt} - \Delta u - \Delta u_t = |u|^{p-2} u \ln |u|, & x \in \Omega, \quad t > 0, \\ u(x,t) = 0, & x \in \partial\Omega, \quad t > 0, \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), & x \in \Omega. \end{cases} \tag{1.1}$$

Here, Ω is a bounded domain of $\mathbb{R}^n$ with a smooth boundary $\partial\Omega$. Additionally, we assume that

$$u_0 \in H_0^1(\Omega), \quad u_1 \in L^2(\Omega), \tag{1.2}$$

and

$$\begin{cases} 2 < p < \frac{2n}{n-2}, & \text{for } n \geq 3, \\ 2 < p < \infty, & \text{for } n = 2. \end{cases} \tag{1.3}$$

The logarithmic nonlinearity appears in many branches of physics such as nuclear physics, optics and geophysics, while the linear strong damping term $-\omega \Delta u_t$ appears in models describing so-called Kelvin-Voigt materials exhibiting both elastic and viscous properties [1].