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ON ASYMPTOTIC STABILITY OF SECOND ORDER DIFFERENTIAL EQUATIONS WITH TWO COMMENSURATE DELAYS

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Abstract. In this paper we study the asymptotic stability of the zero solution of second order linear delay differential equations of the form

$$y''(t) = b_0 y'(t) + b_1 y'(t-\tau) + a_0 y(t) + a_1 y(t-\tau) + a_2 y(t-2\tau)$$

where a_0, a_1, a_2, b_0, and b_1 are constants and $\tau > 0$ is a constant delay. There are no robust stability criteria for the zero solution of this equation. Our goal is to derive practical necessary conditions, stability criteria, and stability regions for certain parameters. In proving our results we make use of Pontryagin's theory for quasi-polynomials.
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Keywords: asymptotic stability, stability criteria, sufficient conditions, delay, commensurate delays, characteristic functions.

1 Introduction

The aim of this paper is to study the asymptotic stability of the zero solution of certain second order delay differential equation with two commensurate delays of the form

$$y''(t) = b_0 y'(t) + b_1 y'(t-\tau) + a_0 y(t) + a_1 y(t-\tau) + a_2 y(t-2\tau) \tag{1.1}$$

where $\tau > 0$, a_0, a_1, a_2, b_0, and b_1 are real constants. In our previous papers [1,2], we considered first order neutral and non-neutral equations with commensurate delays, and in [3, 4] we studied second order delay differential equations with one delay. There is considerable interest in second order delay differential equations [5,6,7,8] due to many applications that involve second order delay differential equations. See [6,7,8,9,10,11]. There are no necessary and sufficient conditions for asymptotic stability with more than one delay. Our goal in this paper is to derive practical stability criteria and practical new necessary conditions which can easily rule out asymptotic stability. See also [12,13] for studies of systems that may shed light on (1.1). These studies on systems do not, however, yield complete practical stability criteria of (1.1). It is clear that with 6 independent parameters in (1.1) one cannot expect to get regions of stability for the general case; however, we were able to