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DYNAMICS OF A COUPLED SYSTEM OF RATIONAL DIFFERENCE EQUATIONS

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Abstract. In this paper, we derive the explicit solution form of the below system of two dimensional nonlinear rational difference equations from order three

$$w_{n+1}=\frac{(w_n+w_{n-1})(z_{n-1}+z_{n-2})}{z_n-\epsilon}+\mu,\ z_{n+1}=\frac{(w_{n-1}+w_{n-2})(z_n+z_{n-1})}{w_n-\mu}+\epsilon,$$

for $n=0,1,....$ The parameters μ and ϵ are arbitrary positive real numbers with initial values $w_0,w_{-1},w_{-2},z_0,z_{-1}$ and z_{-2}. Then, we state the necessary condition for the existence of the periodic solution with period two of this system. In addition, we prove that the nontrivial positive equilibrium point E_0 of this system is not asymptotically stable by using the linearized method. Furthermore, we show that the non-negative critical point E_0 is nonhyperbolic. Then, we investigate and analyze the qualitative behavior of the system by using the solution expression to present the cases in which the solutions are infinity or move to a constant. Finally, we provide some numerical examples to illustrate the effectiveness of theoretical results.

Keywords. periodicity, equilibrium points, stability, difference equation, dynamics.

AMS (MOS) subject classification: 39A10; 39A23; 39A30.

1 Introduction

Recently, the topics of nonlinear difference equations and their systems have a place among articles, since they appear in many scientific areas. However, in the area of studying nonlinear rational difference equations (RDEs), the researchers are interested in obtaining an explicit form of the solutions and discussing the asymptotic stability of its behavior. Sometimes, the researchers focus on investigating the qualitative analysis of the solutions. This is because some types of nonlinear difference equations cannot be solved. For