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TWIN QUADRATIC POLYNOMIAL VECTOR FIELDS IN THE SPACE $\mathbb{C}^3$

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Quadratic polynomial vector fields, singularities, spectra, twin vector fields **Abstract.** In this paper we study quadratic polynomial vector fields on $\mathbb{C}^3$ with 8 isolated singularities. Either two polynomial vector fields share six singularities with the same position and spectra and the remaining two singularities have some relation on their spectra, or two polynomial vector fields share five singularities with the same position and spectra and the remaining three singularities have some other relation on their spectra. Under these conditions we determine the spectra and positions of the remaining singularities. Moreover there exist two three-parametric families of vector fields having the same singular points and for each singular point both vector fields have the same spectrum..

Keywords. Quadratic polynomial vector fields, singularities, spectra, twin vector fields.

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1 Introduction and statement of the results

Consider polynomial vector fields on the affine space $\mathbb{C}^3$. We denote by $\mathcal{P}$ the space of all polynomial vector fields

$$\chi = P(x,y,z)\frac{\partial}{\partial x} + Q(x,y,z)\frac{\partial}{\partial y} + R(x,y,z)\frac{\partial}{\partial z}$$

such that P, Q and R are quadratic. By Bezout's Theorem, a generic element of $\mathcal{P}$ has exactly eight isolated singularities. We denote by $\mathcal{P}_8$ the space of the vector fields in $\mathcal{P}$ that have eight isolated singularities. Since $\chi \in \mathcal{P}_8$ has the maximum number of singularities, the determinant of the linear part of χ at each singular point is nonzero. So the eigenvalues at any singular point are nonzero, i.e. all singular points are non degenerate (see for more details [5]). The space $\mathcal{P}_8$ is endowed with a structure of a complex affine space identifying all the thirty coefficients of the polynomials P, Q and R with a point of $\mathbb{C}^{30}$. This topology in the set $\mathcal{P}_8$ is called the *topology of the coefficients*, and $\mathcal{P}_8$ is an open subset of $\mathcal{P}$.