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BLOW-UP OF SOLUTIONS TO A VISCOELASTIC EULER-BERNOULLI EQUATION WITH NONLOCAL DISSIPATION

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Abstract. The viscoelastic Euler-Bernoulli equation with nonlinear nonlocal dissipation is considered. Under arbitrary positive initial energy, a finite-time blow-up result is proved by the modified concavity method and an example is given. This is a first result to this problem.

Keywords. viscoelastic Euler-Bernoulli equation; initial boundary value problem; nonlocal weak damping; blow-up; modified concavity method

AMS (MOS) subject classification: 35L75, 35B44

1 Introduction

In the past 40 years, more and more scientists in the field of materials and mathematicians have devoted themselves to researches on the theory and application of viscoelastic materials. The viscoelastic Euler-Bernoulli equation is an important equation. The problem of proving the existence and asymptotic behavior of solutions has been studied from old times.

This paper considered with blow-up of solutions to the initial boundary value problem of the viscoelastic Euler-Bernoulli equation

$$\begin{cases} u_{tt} + \Delta^2 u - \int_0^t g(t-s)\Delta^2 u ds + N(||\nabla u||_2^2)u_t = f(u), x \in \Omega, \\ u = \frac{\partial u}{\partial \nu} = 0, x \in \Gamma, \\ u(x,0) = u_0(x), u_t(x,0) = u_1(x), x \in \Omega, \end{cases} \tag{1}$$

where $\Omega \subset R^n$ is a bounded domain with smooth boundary Γ, $N(||\nabla u||_2^2)$ represents nonlocal coefficient in which $||\cdot||_2$ stands for the norm in $L^2(\Omega)$ and $f(s)$ is a continuous function.

Equation (1) with $g = 0$ was derived from the nonlocal plate equation proposed by Lange and Perla Menzala [1], where the following equation was considered

$$u_{tt} + \Delta^2 u + N(||\nabla u||_2^2)u_t = 0 \; in \; R^n. \tag{2}$$

By using Fourier transform and exploring the regularity of initial data with respect to the spatial dimension n, they proved the existence of global classical solutions and algebraic energy decay. Later, Cavalcanti et al [2] studied