

## WELL-POSEDNESS AND DECAY ESTIMATES FOR A PETROVSKY EQUATION WITH A NONLINEAR STRONG DISSIPATION

Naima Louhibi<sup>1</sup> Akram Ben Aissa<sup>2</sup> and Tayeb Lakroumbe<sup>3</sup>

<sup>1</sup>Laboratory of Analysis and Control of Partial Differential Equations, Djillali Liabes University, P. O. Box 89, Sidi Bel Abbes 22000, Algeria

<sup>2</sup>UR Analysis and Control of PDE's, UR 13ES64, Higher Institute of transport and Logistics of Sousse, University of Sousse, Tunisia.

<sup>3</sup>Laboratory of Analysis and Control of Partial Differential Equations, Djillali Liabes University, P. O. Box 89, Sidi Bel Abbes 22000, Algeria.

**Abstract.** A nonlinear Petrovsky equation in a bounded domain with a strong dissipation is considered

$$\partial_{tt}u + \Delta^2 u - \sigma(t)g(\Delta\partial_t u) = 0.$$

The paper concerns the existence of unique solution by using the energy method combined with the Faedo-Galerkin procedure under assumption on dissipation function  $g$ . Furthermore, the asymptotic behaviour of solutions using the multiplied method is shown.

**Keywords.** Well-posedness, general decay, multiplier method, convexity, Petrovsky equation.

**AMS (MOS) subject classification:** 35B40, 35B45, 35L70.

## 1 Introduction

Let  $\Omega$  is a bounded domain in  $\mathbb{R}^n$ , with smooth boundary  $\Gamma$ , let  $u(x, t) = u$ . Initial-boundary value problem for the nonlinear Petrovsky equation is considered

$$\begin{cases} \partial_{tt}u + \Delta^2 u - \sigma(t)g(\Delta\partial_t u) = 0, & x \in \Omega, t \geq 0, \\ u = \Delta u = 0, & x \in \Gamma, t \geq 0, \\ u = u_0(x), \quad u_t = u_1(x) & x \in \Omega, t = 0, \end{cases} \quad (1)$$

where  $(u_0, u_1)$  is the initial data in a suitable function space,  $g$  is real function satisfying some conditions to be specified later and  $\sigma$  is a positive function. In [6], Guesmia considered the following problem

$$\begin{cases} \partial_{tt}u + \Delta^2 u + q(x)u + g(\partial_t u) = 0 & x \in \Omega, t > 0 \\ u = \partial_\nu u = 0 \text{ in} & x \in \Gamma, t > 0 \\ u = u_0(x), \quad u_t = u_1(x), & x \in \Omega, t = 0, \end{cases} \quad (2)$$

where  $g$  is continuous, increasing, satisfying  $g(0) = 0$  and  $q : \Omega \rightarrow \mathbb{R}_+$  is a bounded under suitable growth conditions on  $g$ , decay results for weak, as well as