Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **28 (2021)** 251-268
 http://www.watam.org

IMPULSIVE MILD SOLUTIONS FOR NONLOCAL FRACTIONAL SEMILINEAR DIFFERENTIAL INCLUSION WITH DELAY IN BANACH SPACES

Nawal A. Alsarori[1*] , Ahmed G. Ibrahim[2] and Kirtiwant P. Ghadle[3]

[1,3]Department of Mathematics, Dr. Babasaheb Ambedkar Marathwada University, Aurangabad, 431004(MS), India

[2]Department of Mathematics, Factuly of Science, King Faisal University Alhasa, Saudi Arabia

Abstract. This paper gives existence results for impulsive fractional semilinear differential inclusions with delay involving Caputo derivative in Banach spaces. We are concerned with the case when the linear part generates a semigroup not necessarily compact, and the multivalued function is upper semicontinuous and compact. The methods used throughout the paper range over applications of Hausdorff measure of noncompactness, and multivalued fixed point theorems. Finally, we provide an example to clarify our results.

Keywords. Impulsive fractional differential inclusions; Nonlocal conditions; Fixed point theorems; Mild solutions, Impulsive differential inclusions with delay.

AMS (MOS) subject classification: 34A60, 34B37, 34G10.

1 Introduction

In this paper, we shall be concerned with the following impulsive differential inclusion with nonlocal condition:

$$(Q)\begin{cases} {}^cD^\alpha x(t) \in Ax(t) + F(t, \tau(t)x),\ t \in J = [0, b],\ t \neq t_i, \\ x(t_i^+) = x(t_i) + I_i(x(t_i^-)), i = 1, ..., m, \\ x(t) = \psi(t) - g(x),\ t \in [-r, 0], \end{cases}$$

where ${}^cD^\alpha$ is the Caputo derivative of order α, $A : D(A) \subseteq E \to E$ is the infinitesimal generator of a C_0-semigroup $\{T(t), t \geq 0\}$ on a real separable Banach space E, $F : J \times \Theta \to 2^E$, $\psi : [-r, 0] \to E$, $0 = t_0 < t_1 < \cdots < t_m < t_{m+1} = b$, for every $i = 1, 2, ..., m$, $I_i : E \to E$ impulsive functions which characterize the jump of the solutions at impulse points, $g : \Lambda \to E$, is a function related to the nonlocal condition at the origin and $x(t_i^+), x(t_i^-)$ are the right and left limits of x at the point t_i respectively. Finally, for any $t \in J$, $\tau(t) : \Lambda \to \Theta$ defined by $x(\theta) = x(t + \theta),\ \theta \in [-r, 0],\ x \in \Lambda.$ and $\Theta,\ \Lambda$ will be specified later.

Impulsive differential equations and impulsive differential inclusions have played a significant role in development of modeling impulsive problems in