

FIXED POINT THEOREMS FOR MULTI-VALUED 1-SET CONTRACTIONS UNDER WEAK TOPOLOGY

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Abstract. The aim of this paper is to prove new fixed-point results for weak 1-set contraction multi-valued mappings with respect to some measure of noncompactness. The results are used to obtain some fixed point theorems for the sum of nonlinear operators. To illustrate the applicability of the theoretical result, we supply an example of application concerned with the solvability of a nonlinear integral inclusion.

Keywords. weak 1-set contraction, weak ψ -expansive map, De Blasi measure of noncompactness, fixed point, differential inclusion.

AMS (MOS) subject classification: 26E25, 34A60, 47H08, 47H09, 47H10.

1 Introduction and Preliminaries

Fixed point theory under weak topology features has gained very much attention in the recent literature (see, e.g., [2, 3, 11, 12, 17, 18, 19]). The theory of condensing operators started in 1967 with the pioneer paper of Sadovskii [20]. Sadovskii's result was afterwards improved using different ideas and directions. We quote, for example, Himmelberg, Porter, and Van Vleck [15] who extended the definition of a condensing operator to multi-valued multi-functions and provided a multi-valued version of Sadovskii's theorem. Later, Daher [8] showed that Sadovskii's fixed point theorem still holds for countably condensing maps with respect to some measure of noncompactness. Then multi-valued analogs of Daher fixed point theorem were obtained by Agarwal and O'Regan [18] for classes of upper semicontinuous multimappings. More recently, Cardinali and Rubbioni [7, 6] considered weakly closed graph multimaps and established a multi-valued version of Daher's Theorem for weakly closed graph multimaps (i.e., weakly closed graph with respect to the weak topology) in a separable Banach space.