Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **28 (2021)** 107-132
 http://www.watam.org

POISSON STABILITY AND POINCARÉ RECURRENCE THEOREM FOR IMPULSIVE CONTROL AFFINE SYSTEMS

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Abstract. A state point of a system is Poisson stable if its neighborhoods are recursive with respect to it. This property coincides with the Poincaré recurrence and is satisfied by Lyapunov stable points, periodic points, and controllable points. The present manuscript is dedicated to the Poisson stability for impulsive control affine systems. It presents the characterization of positively Poisson stable point by means of the impulsive positive semi-orbit. The Poisson stability is linked to the notions of recurrence, periodicity, controllability, and nonwandering points. The Poincaré recurrence theorem is reproduced.

Keywords. Impulsive control system; impulsive nonautonomous dynamical system; Poisson stability; Poincaré recurrence; recurrence theorem.

AMS (MOS) subject classification: 93B05; 93D05; 93D20

1 Introduction

In the last twenty years, the stability theory of impulsive systems has been intensive studied. In the Lyapunov sense, many types of stability were characterized by means of Lyapunov functionals (e.g. [7, 14, 20, 21, 24]). The concept of Poisson stability for impulsive semidynamical systems was studied by E. Bonotto and N. Grulha [6]. In the present paper, we extend the studies of Poisson stability to the setting of impulsive control affine systems.

We assume the paradigm of impulsive nonautonomous dynamical system that considers dynamic changes by means of an impulsive set together with an impulsive function ([4, 5, 24]). In the physical sense, the impulsive state is determined by an impulsive function representing some external force that interrupts the evolution of the system by abrupt changes of state. For instance, the billiard-type system can be modeled by differential systems with impulses acting on the first derivatives of the solutions, since the velocities of the colliding balls gain finite increments at the moments of impact. Technically, this formulation enables a description of the impulsive control affine system by means of the impulsive system semigroup ([24]). By following