

EXISTENCE AND CONVERGENCE THEOREMS FOR COMMON ATTRACTIVE POINTS OF A FINITE FAMILY OF FURTHER GENERALIZED HYBRID MAPPINGS IN HILBERT SPACES

Aree Varatechakongka¹ and Withun Phuengrattana^{1,*}

¹Department of Mathematics, Faculty of Science and Technology,
Nakhon Pathom Rajabhat University, Nakhon Pathom 73000, Thailand

*Corresponding author email: withun_ph@yahoo.com

Abstract. In this paper, we establish the common attractive point theorem without the commonly required convexity for a finite family of further generalized hybrid mappings in Hilbert spaces. Moreover, we prove weak and strong convergence theorems of one step iterative method for such a class of nonlinear mappings in Hilbert spaces. Our main results extend and generalize many results in the literature.

Keywords. common attractive point, further generalized hybrid mapping, Hilbert space, quasi-nonexpansive mapping, weak convergence.

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1 Introduction

Let H be a real Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and norm $\|\cdot\|$, and C be a nonempty subset of H . Let T be a mapping of C into itself. Recall that the set of fixed points of T is denoted and defined by $F(T) = \{z \in C : Tz = z\}$. For several years, the study of fixed point theory of nonlinear mappings has attracted, and continues to attract, the interest of several well-known mathematicians (see, e.g., [1, 2, 3]).

In 2011, Takahashi and Takeuchi [4] introduced the concept of attractive points in Hilbert spaces. They defined and denoted the set of attractive points as follows:

$$A(T) = \{z \in H : \|Tx - z\| \leq \|x - z\|\}$$

for all $x \in C$. From this definition, neither an attractive point is a fixed point nor conversely. However, they also gave some properties of the attractive points as follows.

Lemma 1.1 *Let H be a real Hilbert space and C be a nonempty subset of H . Let $T : C \rightarrow C$ be a mapping. Then $A(T)$ is a closed and convex subset of H .*

Lemma 1.2 *Let H be a real Hilbert space and C be a nonempty closed convex subset of H . Let $T : C \rightarrow C$ be a mapping. If $A(T) \neq \emptyset$, then $F(T) \neq \emptyset$.*