Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **26 (2019)** 279-290
 http://www.watam.org

OPTIMAL RELAXED CONTROL FOR A CLASS OF NONLINEAR AND NONCOVEX DYNAMIC SYSTEMS

Shi'an Wang and N.U. Ahmed

School of Electrical Engineering and Computer Science
University of Ottawa, Ottawa, ON K1N 6N5, Canada

Abstract. In this paper we consider optimal relaxed control problem for a class of nonlinear and nonconvex dynamic systems governed by ordinary differential equations. We first formulate the regular control problem and briefly discuss its limitations and then transform it into a more general relaxed control problem. Thereafter, we present some results on existence and uniqueness of solutions and introduce the necessary conditions of optimality using a general minimum principle. Then a computational algorithm is developed to find the optimal switching functions representing the frequencies of usage of distinct control values. Finally, for the purpose of illustration, we present a series of numerical simulation results for two different dynamic systems.

Keywords. Nonlinear dynamic systems, relaxed controls, necessary conditions of optimality, Pontryagin's minimum principle.

AMS (MOS) subject classification: 93C10, 93C15, 49J15, 49K15.

1 Introduction

Control theory has gone through phenomenal development over the past few decades. In addition to its significance in creating theoretical foundations, control theory also plays a pivotal role in a broad range of real-world applications. In recent years control theory has been applied to a great number of research areas, such as computer communication networks [3], transportation systems [9], [11], energy economics [6], operations research [10], etc. As an indispensable part of control theory, optimal control has gained enormous attention from scientific community due to its power in optimal decision making and finding an optimal control law to meet any given objective. Apart from theoretical advancement, there have been many numerical techniques [8] developed for the computation of optimal control problems. In the applications of optimal control theory, there are many practical problems where the control constraint set is nonconvex indicating that there may not exist an optimal solution from the class of measurable functions (called regular controls), and that the classical calculus of variations is not applicable. In order to address such problems, we can extend the class of admissible controls to a larger class known as relaxed controls covering the regular controls as a subclass so that an optimal solution can be found in the sense of probabilistic