Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **26 (2019)** 231-260
 http://www.watam.org

PATTERNS IN A FRESH WATER TUSSOCK SEDGE MODEL WITH TWO LIMIT CYCLES

Jinliang Wang[1], Xiaojie Hou[2] and You Li[1]

[1]LMIB & School of Mathematics and System Science
Beihang University, Beijing 100191, P. R. China

[2]Department of Mathematics and Statistics
University of North Carolina Wilmington, Wilmington NC 28403, USA

Abstract. We study the pattern formation mechanism in a reaction diffusion system arising from the interactions between tussock sedges and wracks. We first show that the corresponding kinetic system has at least two limit cycles, which implies that the tussock sedges grow in a cyclic pattern regardless of the location. Through a center manifold reduction and an application of the Hopf bifurcation theorem, we derive the conditions for the occurrence of the Turing patterns. We find the patterns are of spot and stripe type, which is in agreement with the field observations. Interestingly, the numerical simulations also suggest that there are layered Turing patterns existing in this model, i.e., for some parametric values, a stable equilibrium and a surrounding stable limit cycle experience Turing instability simultaneously. It seems that only a limited number of results on the layered Turing patterns exist in literature.

Keywords. Turing patterns; Hopf bifurcation; Supercritical; Subcritical; Stability

AMS (MOS) subject classification: 35K10, 35K40, 35K57, 35K58

1 Introduction

Tussock sedge grasses can be found abundantly in wetlands and marshes in North America and other places of the world. As a perennial species the grasses can live more than one year. Tussock sedge is an important link in wetland ecosystem. Tussock sedge grasses have long roots that can deeply penetrate into soil, thus they can be used to stabilize slope and soil and improve the porosity of the earth. When the old leaves die the new leaves grow on top of the dead ones and form mounts, which can be observed from far away. Growing in clumps, the space among tussock sedges provides habitats for small animals. A mathematical study of the growth patterns of the tussock sedge is not only interesting in theory but also practical in the real world. From biological point of view, the Tussock sedge grasses and the wracks are essential to the pattern formation process. The dead grasses activate the wracks and the wracks in turn inhibit the growth of the grasses. Patterns are formed due to the interaction of the grasses and the wracks as well as the unequal diffusions of the two. In [1], an experiment site was set up to study the spatial patterns of the tussock sedge and the authors concluded,