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A NOTE ON CHAOS FOR $\mathbb{Z}^D$-ACTION

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Abstract. In this note we define and study the notion of k-type Devaney chaos for a $\mathbb{Z}^d$-action on a compact metric space. We study the relationship between k-type Devaney chaoticity of a $\mathbb{Z}^d$-action on a space X and its induced $\mathbb{Z}^d$-action on the hyperspace $\mathcal{K}(X)$. The notions of k-type mixing and k-type weak mixing are defined and studied. Using these notions, we find conditions for k-type Devaney chaoticity of the induced $\mathbb{Z}^d$-action on the hyperspace.

Keywords. Transitivity, Mixing, Weak-mixing, Sensitive dependence on initial conditions, Devaney Chaos.

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1 Introduction

A numerous class of real problems are modeled by a discrete dynamical system (X, f), where X is a metric space and $f : X \to X$ is a continuous map. The basic goal of the theory of discrete dynamical systems is to understand the nature of orbits $x, f(x), f^2(x), ..., f^n(x),\ x \in X$, as n becomes large. More precisely the study of orbits in a discrete dynamical system reveals how the points move in the base space X. Sometimes, it is not sufficient to know how points are moved in the base space X but it is necessary to know how the subsets of X are moved. This leads to the problem of analyzing the dynamics of the set valued discrete dynamical systems. In [1], Flores posed the following question: What is the relationship between the dynamics of individual movement and the dynamics of collective movement? Since then many researchers have attempted to answer this question. The term chaos in connection with a map was firstly used by Li and Yorke [3] without giving any formal definition. Now there are various definitions of chaos. A common idea of all of them is to show the complexity and unpredictability of the behavior of the orbits of a system.