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PARTIAL STABILITY OF LINEAR STOCHASTIC FUNCTIONAL DIFFERENTIAL EQUATIONS AND THE W-TRANSFORM

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Abstract. The aim of the paper is two-fold. Firstly, we show how partial Lyapunov stability for linear stochastic functional differential equations (SFDE) can be studied via partial input-to-state stability. Secondly, we demonstrate how N.V.Azbelev's W-transformations can be used to find conditions of partial input-to-state stability. As a result, we end up with verifiable criteria of asymptotic and exponential partial Lyapunov stability with respect to the initial values for general and specific SFDE. A number of examples is provided showing efficiency of the method.

Keywords. Partial stability, stochastic differential equations, aftereffect, semimartingales, integral transforms.

AMS (MOS) subject classification: 34K50, 34D20.

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