CHEAP QUADRATIC CONTROL OF LINEAR SYSTEMS WITH STATE AND CONTROL DELAYS

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Abstract. A finite horizon quadratic cheap control of a linear system with point-wise and distributed time delays in the state and control variables is considered. By a linear transformation, this problem is converted to an equivalent linear-quadratic cheap control problem for a much simpler (undelayed) system. For this new problem, asymptotic expansions of the optimal control and the optimal value of the cost functional are constructed. The boundedness of the optimal control is established. The limit of the optimal value of the cost functional is obtained. Based on these results, an asymptotic behavior of solution to the original control problem is analyzed. Illustrative examples are presented.

Keywords. Time delay system, cheap control problem, asymptotic expansion, control boundedness, maximal achievable accuracy.

AMS (MOS) subject classification: 93C23, 49N10, 93C73, 93B05

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