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ON THE LOCAL DYNAMICS OF POLYNOMIAL DIFFERENCE EQUATIONS WITH FADING STOCHASTIC PERTURBATIONS

John Appleby[1], Cónall Kelly[2], Xuerong Mao[3] and Alexandra Rodkina[2]

[1]School of Mathematical Sciences,
Dublin City University, Ireland

[2]Department of Mathematics,
University of the West Indies, Jamaica

[3]Department of Statistics and Modelling Science,
University of Strathclyde, UK

Abstract. We examine the stability-instability behaviour of a polynomial difference equation with state-independent, asymptotically fading stochastic perturbations. We find that the set of initial values can be partitioned into a stability region, an instability region, and a region of unknown dynamics that is in some sense "small". In the first two cases, the dynamic holds with probability at least $1-\gamma$, a value corresponding to the statistical notion of a confidence level. Aspects of an equation with state-dependent perturbations are also treated.

When the perturbations are Gaussian, the difference equation is the Euler-Maruyama discretisation of an Itô-type stochastic differential equation with solutions displaying global a.s. asymptotic stability. The behaviour of any particular solution of the difference equation can be made consistent with the corresponding solution of the differential equation, with probability $1-\gamma$, by choosing the stepsize parameter sufficiently small. We present examples illustrating the relationship between h, γ and the size of the stability region.

Keywords. Nonlinear stochastic difference equation, stability, instability, numerical simulation.

AMS (MOS) subject classification: 39A10, 39A11, 37H10, 34F05, 93E15, 60E05

1 Introduction

The global a.s. asymptotic stability of solutions of nonlinear stochastic difference equations has been widely discussed in the literature. The most relevant publications are: [1, 2, 3, 4, 5, 7, 14, 15]. However, little attention has been paid to local stability for such equations. An early attempt to address local dynamics in an equation with bounded noise can be found in Fraser et al [9]; general results for equations with fading, state independent noise may be found in [1].