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RISK-CONSTRAINED DYNAMIC PORTFOLIO MANAGEMENT

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Abstract. We consider a portfolio problem when a Tail Conditional Expectation constraint is imposed. The financial market is composed of n risky assets driven by geometric Brownian motion and one risk-free asset. The Tail Conditional Expectation is derived, re-calculated at short intervals of time and imposed continuously. The method of Lagrange multipliers is combined with the Hamilton-Jacobi-Bellman equation to insert the constraint into the resolution framework. A numerical method is applied to obtain an approximate solution to the problem. We find that the imposition of the Tail Conditional Expectation constraint when risky assets evolve following a log-normal distribution, curbs investment in the risky assets and diverts the wealth to consumption.

Keywords. Portfolio optimization, Risk management, Value-at-Risk, Power utility, Tail Conditional Expectation.

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1 Introduction

In recent years, particular stress has been laid on the substitution of variance as a risk measure in the standard Markowitz [8] mean-variance problem. Since it makes no distinction between positive and negative deviations from the mean, variance is a good measure of risk only for distributions that are (approximately) symmetric around the mean such as the normal distribution or more generally, elliptical distributions (see e.g., McNeil, Frey and Embrechts [9]). However, in most cases such as in portfolios containing options, as well as credit portfolios, we are dealing with wealth distributions that are highly skewed. It is thus more reasonable to consider asymmetric risk measures since individuals are typically loss averse. In this regard, Value-at-Risk (VaR), a downside risk measure (see, e.g., Jorion [6]), has emerged as the industry standard with regulatory authorities enforcing its use.

Despite its widespread acceptance, VaR is known to possess unappealing features. Artzner et al. [2] proposed an axiomatic foundation for risk measures, by identifying four properties that a reasonable risk measure should